Topics List for Final Exam (Math 95/Schusteff)

Class Handouts (and **§4.1**) ***The Right Triangle Approach to Trigonometry***

Definitions of the Six Trig Functions of an acute angle via side ratios of a right triangle.
Complementary Angles and Cofunctions
Trig functions as side lengths of the three special "unit length" right triangles.
The three basic Pythagorean Trig Identities (corresponding to the triangles above)
Using Trig Functions for Solving "Triangle Problems"

§2.1 ***Angles*** *(Static = Two Rays vs. Dynamic = Rotation of Initial Ray to Terminal Ray)*

Radian measure of Angles, Positive and Negative Angles
Memorize positions of terminal rays for "special angles": $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6} \ldots$
Word Problems Involving angles, angular velocity, and "linear speed".

§2.2 ***The Unit Circle Approach to Trigonometry***

Definitions of the Six Trig Functions in terms of coordinates of a point on Unit Circle.
Know the six trig function values at the "special angles" (= all multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$…see pg. 173)
Finding the Six Trig Function values given a Point on the Terminal Ray of an Angle.

Visualizing the values of Tan and Sec in the Unit Circle Context

Visualizing the values of Cot and Csc in the Unit Circle Context

Be able to "read" each of the six trig function values from a "circle diagram with tick marks", as in the "Trig Functions by Hand" exercises handout given in class (see PDF on class web site a few lines underneath the line with HW from Section 2.3).

§2.3—2.6 ***Properties of Trigonometric Functions and their Graphs***

Know the shapes of the graphs for the six basic trigonometric functions, and the various aspects related to those shapes including:

- domains and ranges
- period
- amplitude
- horizontal intercepts and/or vertical asymptotes
- location of maxima and minima
- intervals of increase and intervals of decrease
- relationship of all the above to the "key points" corresponding to inputs occurring at the "quarter-points" of any given period.

[Note: See pg 174 of our text for a visual summary of the above for the six trig functions.]

Be able to graph an equation of the form: $y = A\,"trig"(\omega x - \varphi)$, where "trig" denotes any one of the six trig functions, using information summarized above.

Understand how the graph of an equation of the form: $y = A\,"trig"(\omega x - \varphi)$, is obtained from the graph of $y = "trig"(x)$ via the following sequence of geometric transformations of the plane:

1. Vertically scale the graph of $y = "trig"(x)$ by A, to obtain the graph of $y = A\,"trig"(x)$
2. Horizontally scale the graph of $y = A\,"trig"(x)$ by $c = \frac{1}{\omega}$, to obtain the graph of $y = A\,"trig"(\omega x)$

3. Horizontally shift the graph of $y = A\,\text{"}trig\text{"}(\omega x)$ by $s = \dfrac{\varphi}{\omega}$, to obtain the graph of

$$y = A\,\text{"}trig\text{"}\left(\frac{1}{c}(x-s)\right) \quad \text{, or equivalently:} \quad y = A\,\text{"}trig\text{"}\left(\omega(x - \frac{\varphi}{\omega})\right) = A\,\text{"}trig\text{"}\left(\omega x - \varphi\right) \quad .$$

Be able to recognize which trig function corresponds to a given graph.
Be able to read period, amplitude, and phase shift info from a given trig graph.
Be able to find the equation of a scaled and shifted trig function.

1.7 ***Inverse Functions and their Graphs***

Understand and be able to use basic relationships below which hold for a function and its inverse.

$$\begin{matrix} Dom(f^{-1}) = Range(f) \\ Range(f^{-1}) = Dom(f) \end{matrix} \quad \text{and} \quad \begin{matrix} f^{-1}(f(x)) = x \\ f(f^{-1}(x)) = x \end{matrix} \quad \text{for all} \quad \begin{matrix} x \in Dom(f) \\ x \in Dom(f^{-1}) \end{matrix}$$

The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are reflections of one another in the line $y = x$.
Given the formula for a function $f(x)$, be able to find the formula for its inverse $f^{-1}(x)$.
Understand and be able to use basic relationships which hold for a function and its inverse.

§3.1-3.2 ***Inverse Trig Functions***

Know the definitions and domains and ranges of the six inverse trigonometric functions:

Domain of $\sin^{-1}, \cos^{-1} = [-1, 1]$ Range of $\sin^{-1}, \tan^{-1}, \csc^{-1} = [-\frac{\pi}{2}, \frac{\pi}{2}]$ or $(-\frac{\pi}{2}, \frac{\pi}{2})$
Domain of $\tan^{-1}, \cot^{-1} = (-\infty, \infty)$ Range of $\cos^{-1}, \cot^{-1}, \sec^{-1} = [0, \pi]$ or $(0, \pi)$
Domain of $\sec^{-1}, \csc^{-1} = (-\infty, 1] \bigcup\ [1, \infty)$

Know how to evaluate inverse trig functions (especially for inputs yielding the "special angles")
Know how to use trig identities and/or triangle diagrams to evaluate expressions of the form:
Trig(ArcTrig(x)) …for example: $\tan(\sin^{-1}(x))$

§3.3 ***Solving Trigonometric Equations***

Be able to use algebra and appropriate trig identities to solve trigonometric equations.
The basic steps are: 1) Rewrite given equation in form $\sin(\theta) = number$, or $\cos(\theta) = number$
2) Draw appropriate circle diagrams to find all solution values of θ.

§3.4 ***Trigonometric Identities***

Be familiar with the basic trig identities:

Quotient Identities: $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$, $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$

Reciprocal Id's: $\sec(\theta) = \dfrac{1}{\cos(\theta)}$, $\csc(\theta) = \dfrac{1}{\sin(\theta)}$, $\cot(\theta) = \dfrac{1}{\tan(\theta)}$, $\tan(\theta) = \dfrac{1}{\cot(\theta)}$

Pythagorean Id's: $\cos^2(\theta) + \sin^2(\theta) = 1$, $1 + \tan^2(\theta) = \sec^2(\theta)$, $\cot^2(\theta) + 1 = \csc^2(\theta)$

& "variations": $\cos^2(\theta) = 1 - \sin^2(\theta)$, $\sin^2(\theta) = 1 - \cos^2(\theta)$, $\tan^2(\theta) = \sec^2(\theta) - 1$

Even/Odd Id's: $\cos(-\theta) = \cos(\theta)$, $\sin(-\theta) = -\sin(\theta)$, $\tan(-\theta) = -\tan(\theta)$

Be able to write down steps and do the appropriate algebra needed to verify a given trig identity.

§3.5 *Angle Sum and Difference Formulas*

Know and be able to use the angle sum/difference formulas for sine, cosine, and tangent:

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \qquad \tan(\alpha \pm \beta) = \frac{\tan(\alpha)) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}.$$

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$$

§3.6 *Double Formulas*

Know and be able to use the double and half-angle formulas for sine, cosine, and tangent:

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\beta) \qquad \tan(2\alpha) = \frac{2\tan(\alpha)}{1 - \tan^2(\alpha)}$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \qquad \cos(2\alpha) = 1 - 2\sin^2(\alpha) \qquad \cos(2\alpha) = 2\cos^2(\alpha) - 1$$

§4.1-4.3 Solving Triangle Problems

Given various side lengths and/or angles in a triangle, be able to find the other side lengths & angles:
§4.1: …using Pythagorean Theorem, and basic definitions of trig & inverse trig functions for right triangles;
§4.2: …using the Law of Sines for the cases ASA or AAS for non-right triangles; or
§4.3: …using the Law of Cosines for the cases SAS or SSS for non-right triangles.